Michelson Interferometry of High-order Harmonic Generation in Solids

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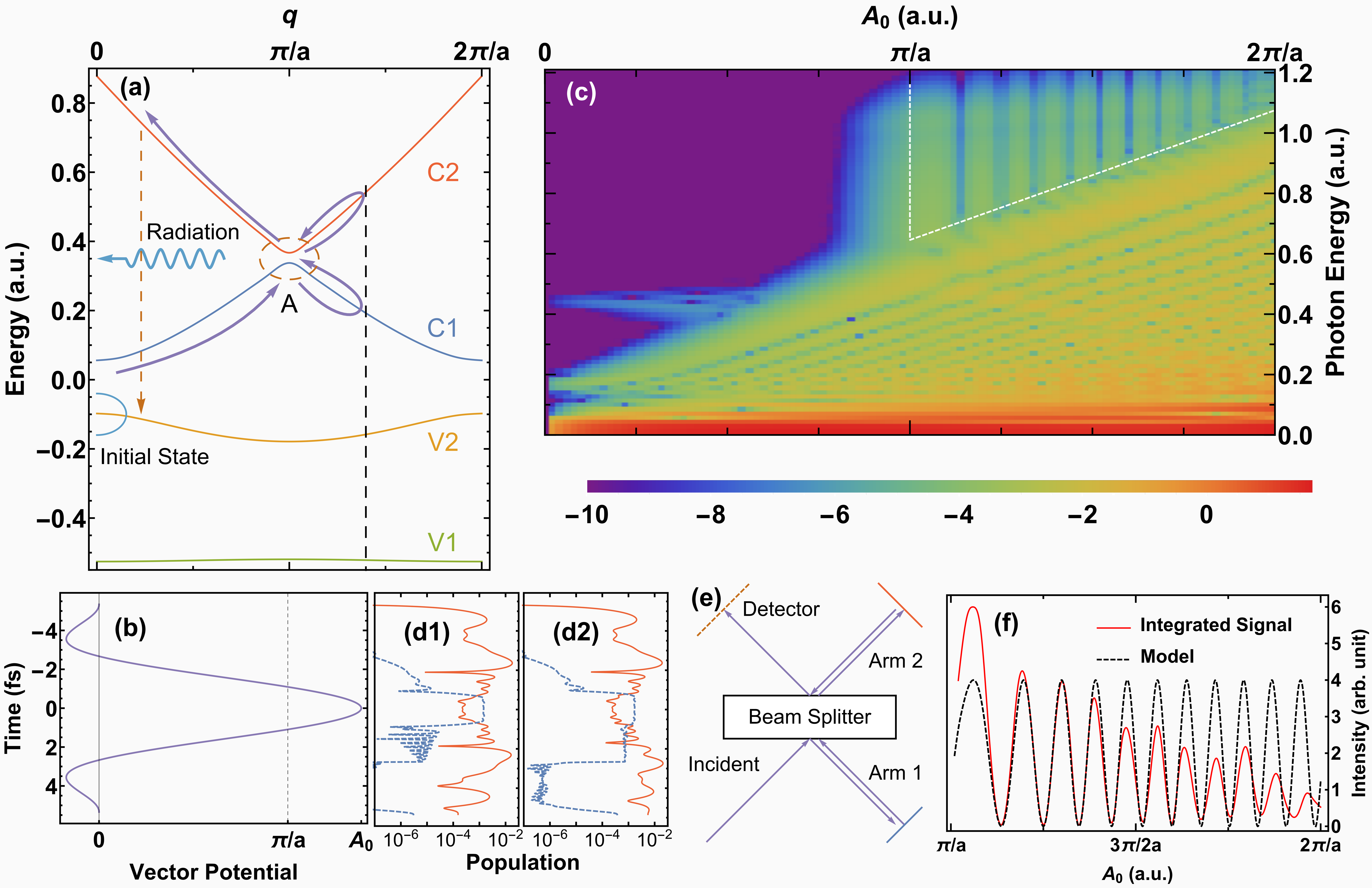

Figure 1: The realization of the Michelson interferometer of Bloch electron in the HHG spectra. For the model with the first four bands shown in (a), the harmonic spectra are shown in (c) plotted in the logarithmic scale as a function of A_0 of the pulse, as shown in (b). The Michelson interferometry is present at the top-right corner of (c), enclosed by the white dotted lines. The Bloch electron Michelson interferometer is in the same principle with the usual optical one, as sketched in (e). The interferences minimum and maximum can be verified by checking the populations of C1 (solid red line) and C2 (dashed blue line) as a function of time, as respectively shown in (d1) for $A_{01} = 1.14\pi/a$ and (d2) for $A_{02} = 1.18\pi/a$. The interference patterns can be successfully be explained by the analytical model of Eq. (6), as shown in (f).

Background knowledge

Within single-active electron approximation and dipole approximation, solid system interacting with laser field is govern by

$$\mathrm{i}\partial_t \Psi(r,t) = \left\{\frac{1}{2}[\hat{p} + A(t)]^2 + V(r)\right\}\Psi(r,t), \tag{1}$$

According to Bloch theorem, $\Psi(r,t)$ can be expressed as $\exp(\mathrm{i}q\cdot r)u(r,t)$, where $u(r,t)$ is a periodic function which satisfied

$$\mathrm{i}\partial_t \Psi(r,t) = \left\{\frac{1}{2}[\hat{p} + q + A(t)]^2 + V(r)\right\}\Psi(r,t), \tag{2}$$

If $A(t)$ varying slow enough, the total Hamiltonian adiabatically changes as the field-free Hamiltonian with a time-dependent parameter $q(t) = q_0 + A(t)$. According to the quantum adiabatic theorem, the population in each eigenstate is essentially unchanged

$$|\Psi(t)\rangle \approx \exp\left[-\mathrm{i}\int_{t_0}^{t}\varepsilon_n(q(\tau))\,\mathrm{d}\tau\right]|n(q(t_0))\rangle\,, \tag{3}$$

where

$$H_{\text{free}}(q)\,|n(q)\rangle = \varepsilon_n(q)\,|n(q)\rangle\,. \tag{4}$$

This approximation cannot be made if the energy difference of two eigenstates is not much larger than the change speed of $q(t)$, in which case the Landau-Zener tunneling mostly occurs.

Numerical experiment

We use a simple 1D model to demonstrate our proposal. Periodic potential is taken to be

$$V(x) = V_0\left(1 + \cos\frac{2\pi x}{a}\right), \tag{5}$$

with parameters $V_0 = -0.37\,\text{a.u.}$, $a = 8\,\text{a.u.}$. A half-cycle pulse which show in Fig.2(b) is applied to induce HHG. Crank-Niclson method is used to solve the TDSE.

Anlynatic model and discussion

The electron starts from the initial state in V2, is then excited to C1, and with the increase of the vector potential it can move towards the avoided crossing point 'A'. It is then split into two parts, one part keeps to move along C1 and the other moves along C2, which corresponds to a population partition between the two bands. Similar to optical Michealson interferometer, electrons move along different paths will accumulate a difference of phase, which will affect the population remaining in C2. In the end, it will determine the yield of harmonic signal Y

$$Y \propto 1 + \cos\Delta\varphi, \tag{6}$$

with

$$\Delta\varphi = \int_{t_i}^{t_f}[\varepsilon_4(A(t)) - \varepsilon_3(A(t))]\,\mathrm{d}t + (\text{tunneling phase}). \tag{7}$$

The last term can be determined by considering the tunneling process precisely [1].

$$(\text{tunneling phase}) = -\frac{\pi}{2} + 2\delta(\ln\delta - 1) + 2\arg\Gamma(1 - \mathrm{i}\delta), \tag{8}$$

in which $\delta = \Delta^2 a/24\pi|E|$, where Δ is the band gap between the two bands and E is the electric field strength at the tunneling point.

We expect that, by adding another orthogonal pulse after the electron wavepacket splitting, the Berry phase [2] along this closed orbit in the reciprocal space may be measured.

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Reference

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